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Review of PhD thesis of MSc. Mojtaba Biglar

**Strain state in neighbourhood of defects in piezo-electric ceramic material**

## 1. Preliminary notes

The thesis is in the scope of the crack mechanics discipline and it covers the initiation, development and spread of defects. The boundary element method (BEM) is chosen to mathematical analysis of physical phenomena. The object of analysis is the piezoelectric material, which is in the form of the multilayers composite and it constitutes an mechanical actuator.

## 2. Aim, thesis and scope of the dissertation

The aim of the dissertation is the micro-mechanical analysis of failure and defect analysis in piezoelectric actuator. To do this the following tasks should be done.

1. Intergranular crack initiation should be identified.

The weakest element of this material are surfaces among the grains. Based on the cohesive low, it is possible to explain an inter-granular crack initiation. This law is also suitable in order to explain the crack initiation between layers with different mechanical properties (aktuator).

2. The crack evaluation ought to be described.

The constitutive equations of the piezoelectric material is the base of the mathematical description of the crack evaluation. Applying deformation relations, the equilibrium equations are derived.

3. To solve crack equations, the numerical methods should be chosen.

For this purpose the BEM is chosen. In order to perform BEM numerical analysis of development and spread of defects, the precise geometrical information about grains boundaries and surfaces ought to be known.

4. The sample of the piezoelectric material ought to be prepared.

As the piezoelectric material, the barium titanate is chosen. Appropriate geometrical data is achieved via the SEM image. (Scanning Electron Microscope).

The achievement of other goals is the PhD student's own achievements and they are enumerated in section 4. Scientifics achievements.

**3. Essential opinion**

The intergranular crack initiation, its evaluation and the failure appear in grain's boundary more often than on the grain surface. So, the problem of crack initiation and its evaluation is very important problem. The description of this problem is the main subject of the dissertation. To solve this problem, an analysis of mechanical behaviour of grains in the boundary is performed.

To solve the above problem, the BEM is applied. This boundary is described for both metallic interface and ceramic grains. To do this, the SEM image is applied for having the precise geometrical information about boundaries and surfaces. The geometry is given in two dimensional coordinates and geometry points are considered as nodes of elements of BEM analysis.

The numerical solution of the boundary problem via the BEM is the PhD student's own achievements and they are enumerated in section 4. Scientifics achievements. This stage of the dissertation has non errors and it sounds like scientific work.

**4. Scientifics achievements**

The given fracture mechanics equations and the numerical BEM method for solving the boundary problem are well known. Thus, the dissertation belongs to the group of research works in which it is searched a new, practical areas of application of known equations and known numerical methods.

Nevertheless, the solution to the boundary problem of the crack mechanics of piezoelectric materials has required from PhD student to make some scientific contributions, which are enumerated below.

1. Preparing the discrete numerical model in CAD software, in which geometrical quantities about grains boundaries and surfaces from the SEM image have been obtained, so this date is real.
2. Supplementing the numerical algorithm in order to applying BEM for separate grains or groups of grains they can be considered as anisotropic parts.
3. Working out an adaptive algorithm to consider separate grain boundaries and micro-cracks.
4. Establishing convergence of the solution for successive interactions.

**5. Editorial and formal evaluation of the dissertation**

It seems that the Eq. (3.1) is wrong; where are terms: $u_{k,li}$ and $\varphi_{k,li}$ or in another forms second derivatives of mechanical displacement and electric potential in final equation ? The correct formulations may be found anywhere, for example:

Hui FAN, Kam-Yim SZE, Wei YANG, *Two dimensional contact on a piezoelectric half-space*, Int. J. Solids Structures, **33**, 9, 1305-1315, 1996.

A piezoelectric half-space under contact loading is considered in the present paper. In the following sections, we will focus on a linear piezoelectric material whose constitutive equation is given by:

$$\sigma = C\gamma - eE, \quad D = e\gamma + \varepsilon E. \tag{1}$$

where **C** is the elasticity tensor of rank four, $\varepsilon$ the permittivity tensor of rank two and **e** the piezoelectricity tensor of rank three. When the piezoelectricity vanishes, the problem decouples into an anisotropic elastic and a dielectric problem.

2. STROH'S FORMULATION FOR PIEZOELECTRIC MATERIALS

In rectangular coordinates the linear piezoelectric solid is described by:
*Constitutive laws*:

$$\sigma_{ij} = C_{ijkl}\gamma_{kl} - e_{kij}E_k$$
$$D_i = e_{ikl}\gamma_{kl} + \varepsilon_{ik}E_k \qquad (2)$$

where $\sigma_{ij}$, $\gamma_{ij}$, $D_i$ and $E_k$ are stress, strain, electric displacement (or electric induction) and electric field, respectively.

*Deformation relations*:

$$\gamma_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k}),$$
$$E_k = -\varphi_{,k}, \qquad (3)$$

where $u_k$ and $\varphi$ are mechanical displacement and electric potential, respectively.

*Equilibrium equations*:

$$\sigma_{ij,i} = 0,$$
$$D_{i,i} = 0, \qquad (4)$$

provided body force and electric source are absent. Substituting eqns (2) and (3) into eqn (4) yields:

$$(C_{ijkl}u_k + e_{lji}\varphi)_{,li} = 0$$
$$(e_{ikl}u_k - \varepsilon_{il}\varphi)_{,li} = 0. \qquad (5)$$

Note, that the Eqs. (5) above are the modal equations yet.

Major editorial and formal errors, examples:

- ... to Eq. (3.1): In these equations, $\sigma$, $D$, $u$ and $\varphi$ are stress, electric displacement, displacement and $u_\alpha$ respectively. How can this be interpreted ?

1.3.

- Take stress –strain curve as an example ... ought to be: The stress –strain equation ....
- ... cohesive law is suitable in order to model interface where ... ought to be: ... to explain the crack initiation ...
- ... Caushy stress tensor ?, favourite methods ?, loosing the generality ?

Arrangement of the content, division on chapters, formulation of the aim are acceptable and final conclusions are transparent. The choice of references is appropriate and representative to the field of knowledge covering the issues analyzed in the dissertation.

## 6. Critical remarks

As can be seen in Eq. (3.17), fundamental solutions are singular. Next, they are constitute integrands in the integral solution given by Eq. (3.18).

**How did PhD student overcome these singularities ?**

**Why did PhD student choose singular BEM in the classic formulation boundary problem?** In next researches I propose unsingular BEM, i.e. in the form of the series with the base in the form of Trefftz functions or Kupradze ones. Furthermore the problem ought to be rewrite in original or inverse variational formulations.

The crack surface is marked via $\Gamma$ in Fig. 3.2. How PhD student has integrated in Eq. (3.31) over the singular $\Gamma$?

... above Eq. (4.1): "The modified boundary element formulation is similar to one which was presented in previous chapter ..." No ! In Eq. (4.1) there is an integral over volume of the domain and this term should be derived in different manner.

... under Eq. (4.1); concern the $E_{ijk}$; fundamental solution and Green function is not the same !

Critical remarks point out areas which PhD student should pay attention if the subject should be continue. These remarks are not arguments disparaging his achievements.

## 7. Summary

PhD student has showed the ability to correctly select and formulate the scientific aim. Next, logically and consistently, with good knowledge of the subject, he has achieved this goal. He has published his knowledge in the monograph and in four articles as co-author. All his works are scientific in nature. So I propose to confer him a Doctor of Philosophy (PhD).